

Geometry and Mensurations-Key Points

Geometry and Mensurations

Mensuration: Mensuration is the branch of mathematics which deals with the study of Geometric shapes , Their area , Volume and different parameters in geometric objects.

Some important mensuration formulas are:

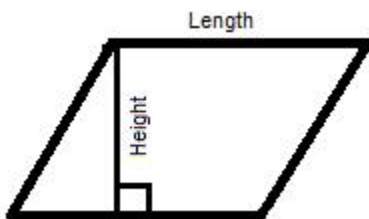
1.Area of rectangle (A) = length(l) * Breath(b);

2.Perimeter of a rectangle (P) = 2 * (Length(l) + Breath (b))

3.Area of a square (A) = Length (l) * Length (l)

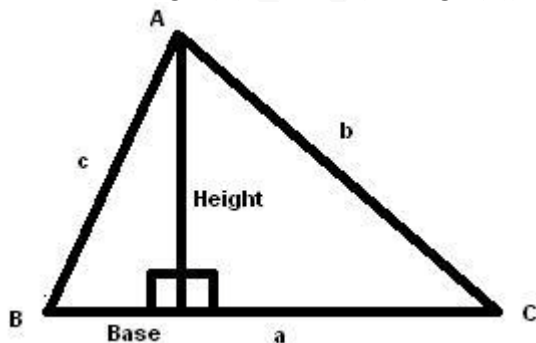
4.Perimeter of a square (P) = 4 * Length (l)

5.Area of a parallelogram(A) = Length(l) * Height(h)



6.Perimeter of a parallelogram (P) = 2 * (length(l) + Breadth(b))

7.Area of a triangle (A) = (Base(b) * Height(b)) / 2



And for a triangle with sides measuring "a" , "b" and "c" , Perimeter = a+b+c

and s = semi perimeter = perimeter / 2 = (a+b+c)/2

And also . Area of triangle,

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

This formulas is also knows as “Hero’s formula”.

8.Area of triangle(A)

$$= \frac{1}{2}a \times b \times \angle C = \frac{1}{2}b \times c \times \angle A = \frac{1}{2}a \times c \times \angle B$$

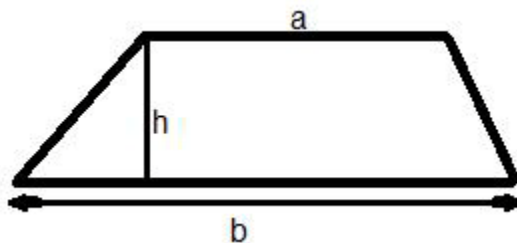
9. Area of isosceles triangle =

$$\frac{b}{4}\sqrt{4a^2 - b^2}$$

Where , a = length of two equal side , b= length of base of isosceles triangle.

10.Area of trapezium (A) =(a+b)/2

Where , “a” and “b” are the length of parallel sides.



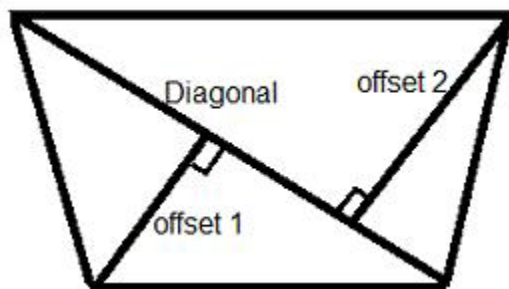
11.Perimeter of a trapezium (P) = sum of all sides

12.Area f rhombus (A) = Product of diagonals / 2

13.Perimeter of a rhombus (P) = 4 * l

where l = length of a side

14.Area of quadrilateral (A) = 1/2 * Diagonal * (Sum of offsets)



15. Area of a Kite (A) = $1/2$ * product of it's diagonals

16. Perimeter of a Kite (A) = 2 * Sum on non-adjacent sides

17. Area of a Circle (A) =

πr^2 , where , r= radius of the circle

18. Circumference of a Circle =

$$2\pi r = \pi d$$

r= radius of circle

d= diameter of circle

19. Total surface area of

cuboid = $2(lb+bh+lh)$

where , l= length , b=breadth , h=height

20. Total surface area of

cuboid = $6l^2$

where , l= length

21. length of diagonal of cuboid =

$$\sqrt{l^2 + b^2 + h^2}$$

22. length of diagonal of cube =

$$\sqrt{3}l$$

23. Volume of cuboid = $l * b * h$

24. Volume of cube = $l * l * l$

25. Area of base of a cone =

$$\pi r^2$$

26. Curved surface area of a cone =C

Where , r = radius of base , l = slanting height of cone

27. Total surface area of a cone =

$$\pi r(r + l)$$

28. Volume of right circular cone =

$$\frac{1}{3} \pi r^2 h$$

Where , r = radius of base of cone , h= height of the cone (perpendicular to base)

29. Surface area of triangular prism = (P * height) + (2 * area of triangle)

Where , p = perimeter of base

30. Surface area of polygonal prism = (Perimeter of base * height) + (Area of polygonal base * 2)

31. Lateral surface area of prism = Perimeter of base * height

32. Volume of Triangular prism = Area of the triangular base * height

33. Curved surface area of a cylinder =

$$2\pi r h$$

Where , r = radius of base , h = height of cylinder

34. Total surface area of a cylinder =

$$2\pi r(r + h)$$

35. Volume of a cylinder =

$$\pi r^2 h$$

36. Surface area of sphere =

$$4\pi r^2 = \pi d^2$$

where , r= radius of sphere , d= diameter of sphere

37. Volume of a sphere =

$$\frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$$

38. Volume of hollow cylinder =

$$\pi r h(R^2 - r^2)$$

where , R = radius of cylinder , r= radius of hollow , h = height of cylinder

39. Surface area of a right square pyramid =

$$a\sqrt{4b^2 - a^2}$$

Where , a = length of base , b= length of equal side ;

of the isosceles triangle forming the slanting face.

40. Volume of a right square pyramid =

$$\frac{1}{2} \times \text{base area} \times \text{height}$$

41. Area of a regular hexagon =

$$\frac{3\sqrt{3}a^2}{2}$$

42. area of equilateral triangle =

$$\frac{\sqrt{3}}{4}a^2$$

43. Curved surface area of a Frustums =

$$\pi h(r_1 + r_2)$$

44. Total surface area of a Frustums =

$$\pi(r_1^2 + h(r_1 + r_2) + r_2^2)$$

45. Curved surface area of a Hemisphere =

$$2\pi r^2$$

46. Total surface area of a Hemisphere =

$$3\pi r^2$$

47. Volume of a Hemisphere =

$$\frac{2}{3}\pi r^3 = \frac{1}{12}\pi d^3$$

48. Area of sector of a circle =

$$\frac{\theta r^2 \pi}{360}$$

where, θ = measure of angle of the sector, r = radius of the sector

Exercise questions

1. What is the area of an equilateral triangle of side 16cm ?

- a) 243 cm²
- b) 64 3 cm²
- c) 363 cm²
- d) 323 cm²

2. Consider the following figure. $\angle A = x + 30$; $\angle D = x - 40$. Find $\angle B$?

- a. 125o
- b. 55o
- c. 155o
- d. 122o

3. Find the area of a square, the product of whose diagonals is 66 cm²

- a) 30 cm²
- b) 33 cm²
- c) 36 cm²
- d) 42 cm²

4. A 5 cubic centimeter cube is painted on all its side. If it is sliced into 1 cubic centimeter cubes, how many 1 cubic centimeter cubes will have exactly one of their sides painted?

- a) 9
- b) 61
- c) 98
- d) 54

5. Find the area of a trapezium whose parallel sides are 20 cm and 18 cm long, and the distance between them is 15 cm.

- a) 225 cm²
- b) 275 cm²
- c) 285 cm²
- d) 315 cm²

6. Examine the figure. $\angle ADB = 25^\circ$; Find $\angle OBC$:

- a. 115°
- b. 25°
- c. 50°
- d. 65°

7. The sector of a circle has radius of 21 cm and central angle 135°. Find its perimeter.

- a) 91.5 cm
- b) 93.5 cm
- c) 94.5 cm
- d) 92.5 cm

8. The volumes of two cones are in the ratio of 1 : 10 and the radii of the cones are in the ratio of 1 : 2, what is the ratio of their vertical heights?

- a) 2 : 5
- b) 1 : 5
- c) 3 : 5
- d) 4 : 5

Answer Key

1.c; 2.b; 3.b; 4.d; 5.c; 6.a; 7.a; 8.a

Exercise Questions

1. If each side of a square is increased by 50%, the ratio of the area of the resulting square to the area of the given square is:

- a. 5:4
- b. 9:4
- c. 4:5
- d. 4:9

2. A man walking at the speed of 4 kmph crosses a square field diagonally in 3 minutes. The area of the field is:

- a. 18000m²
- b. 20000m²
- c. 19000m²
- d. 25000m²

3. The cost of cultivating a square field at the rate of Rs.135 per hectare is Rs.1215. The cost of putting a fence around it at the rate of 75 paise per meter would be :
- a. Rs.360 b. Rs.810 c. Rs.900 d. Rs.1800
4. The cost of carpeting a room 18m long with a carpet 75cm wide at 45 paise per meter is Rs.81. The breadth of the room is:
- a. 7m b. 7.5m c. 8m d. 8.5m
5. A hall 36m long and 15m broad is to be paved with stones, each measuring 6dm by 5 dm. The number of stones required is:
- a. 180 b. 1800 c. 18 d. 18000
6. A room 5.44m long and 3.74m broad is to be paved with square tiles. The least number of square tiles required to cover the floor is:
- a. 176 b. 192 c. 184 d. 162
7. A man cycles round the boundary of a rectangular park at the rate of 12 kmph and completes one full round in 8 minutes. If the ratio between the length and breadth of the park be 3:2, then its area is:
- a. 1536m^2 b. 15360m^2 c. 153600m^2 d. None of these
8. A rectangular carpet has an area of 60 sq.m. If its diagonal and longer side together equal 5 times the shorter side, the length of the carpet is:
- a. 5m b. 12m c. 13m d. 14.5m
9. The cost of papering the four walls of a room is Rs.475. Each one of the length, breadth and height of another room is double that of this room. The cost of papering the walls of this new room is:
- a. Rs.950 b. Rs.1425 c. Rs.1900 d. Rs.712.50
10. The height of a room to its semi-perimeter is 2:5. It costs Rs.260 to paper the walls of the room with paper 50cm wide at Rs.2 per meter allowing an area of 15 sq.m for doors and windows. The height of the room is:
- a. 2.6m b. 3.9m c. 4m d. 4.2m
11. The cross section of a canal is trapezium in shape. The canal is 12m wide at the top and 8m wide at the bottom. If the area of the cross section is 840 sq.m, the depth of the canal is:
- a. 42m b. 84m c. 63m d. 8.75m

12. The altitude of an equilateral triangle of side $3\sqrt{3}$ cm is:

- a. 3cm b. 23cm c. 4.5cm d. $\frac{3}{4}$ cm

13. The area of a right-angled triangle is 30 sq.cm and the length of its hypotenuse is 13cm. The length of the shorter leg is:

- a. 4cm b. 5cm c. 6cm d. 7cm

14. The difference between the circumference and the radius of a circle is 37 cm. The area of the circle is:

- a. 111 cm^2 b. 148 cm^2 c. 154 cm^2 d. 259 cm^2

15. A circular road runs round a circular ground. If the difference between the circumferences of the outer circle and inner circle is 66 metres, the width of the road is:

- a. 5.25m b. 7m c. 10.5m d. 21m

16. A toothed wheel of diameter 50cm is attached to a smaller wheel of diameter 30cm. How many revolutions will the smaller wheel make when the larger one makes 15 revolutions?

- a. 18 b. 20 c. 25 d. 30

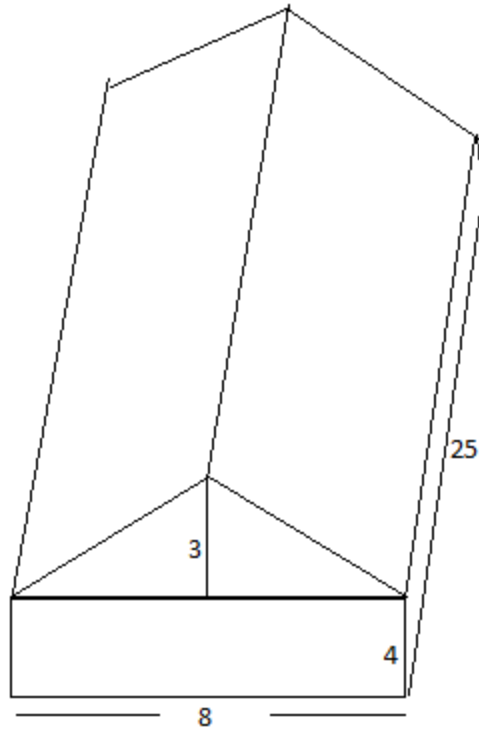
17. A circular wire of radius 42cm is cut and bent into the form of a rectangle whose sides are in the ratio of 6:5. The smaller side of the rectangle is:

- a. 30cm b. 60cm c. 72cm d. 132cm.

18. Four horses are tethered at four corners of a square plot of side 63 metres so that they just cannot reach one another. The area left ungrazed is:

- a. 675.5 m^2 b. 780.6 m^2 c. 785.8 m^2 d. 850.5 m^2

19. A theater is of the shape as shown below. The cross section is a rectangle $8 \text{ m} \times 4 \text{ m}$ mounted by a triangle of altitude 3m. If the length of the building is 25m, find its volume (Inner measures are given).



- a. 1100cm^3 b. 1110cm^3 c. 1010cm^3 d. None of these

20. The measurement of a rectangular box with lid is $25\text{cm} \times 12\text{cm} \times 18\text{cm}$. Find the volume of the largest sphere that can be inscribed in the box (in terms of πcm^3). (Hint: The lowest measure of rectangular box represents the diameter of the largest sphere)

- a. 288 b. 48 c. 72 d. 864

Answer & Explanations:

1. Ans:b

Let, each side= a . Then, original area= a^2 .

New side= $150a/100 = 3a/2$. New area= $9a^2/4$.

$$\text{Required ratio} = 9a^2/4:a^2 = 9:4$$

2. Ans: b.

Length of the diagonal = Distance covered in 3 min. at 4 km/hr.

$$= (4000/60 * 3) = 200\text{m.}$$

Therefore, Area of the field = $1/2 * \text{diagonal}^2$

$$= \frac{1}{2} * 200 * 200 = 20000 \text{ m}^2$$

3. Ans: c

Area = Total cost / Rate = $(1215/135)$ hectares = $(9 * 10000)$ sq.m.

Therefore, side of the square = $\sqrt{90000} = 300\text{m.}$

Perimeter of the field = $(300 * 4)\text{m} = 1200\text{m}$

Cost of fencing = Rs. $(1200 * 3/4) = \text{Rs. } 900$

4. Ans: b

Length of the carpet = Total cost / rate/m = $8100/45 = 180\text{m}$

Area of the carpet = $180 * 75/100 = 135\text{m}^2$

Breadth of the room = $(\text{Area} / \text{length}) = 135/18 = 7.5\text{m}$

5. Ans: b

Area of the hall = $3600 * 1500$

Area of each stone = $(60 * 50)$

Therefore, number of stones = $(3600 * 1500 / 60 * 50) = 1800$

6. Ans: a

Area of the room = $544 * 374 \text{ sq.cm}$

Size of largest square tile = HCF of 544 cm & 374cm

Area of 1 tile = $34 * 34 \text{ sq.cm}$

Therefore, number of tiles = $(544 * 374 / 34 * 34) = 176$

7. Ans:c

Perimeter= Distance covered in 8 min

$$= (12000/60 * 8)m = 1600m$$

Let, length= $3x$ meters and breadth= $2x$ meters

$$\text{Then, } 2(3x+2x) = 1600 \text{ or } x = 160$$

Therefore, length= 480 m and breadth= 320m

$$\text{Therefore, area} = (480 * 320)m^2 = 153600 m^2$$

8. Ans: b

Let, length= x meters and breadth= y meters

$$\text{Then } xy=60 \text{ and } \sqrt{x^2+y^2} + x = 5$$

$$\text{Therefore, } x=60 \text{ and } (x^2+y^2) = (5y-x)^2$$

$$\text{Or } xy=60 \text{ and } 24y^2 - 10xy = 0.$$

$$\text{Therefore, } 24y^2 - 10 * 60 = 0 \text{ or } y^2 = 25 \text{ or } y = 5.$$

Therefore, $x = (60/5)m = 12m$. So, length of the carpet = 12m

9. Ans:c

Let the dimensions of former room be x, y and z .

Then, the area of its 4 walls= $2(x+y)*z$ sq. units.

Dimensions of another room are $2x, 2y$ and $2z$ units.

$$\text{Therefore, area of 4 walls of this room} = 2(2x+2)*2z = 4*[2(x+y)*z]$$

$$= 4 \text{ (Area of 4 walls of 1st room)}$$

$$\text{Therefore, required cost} = \text{Rs.}(475*4) = \text{Rs.}1900$$

10. Ans:c

Let, height= $2x$ metres & (length+ breadth)= $5x$ metres.

Length of paper = $(260/2)m = 130m$.

Therefore, area of paper = $(130*50/100) = 65m^2$

Area of 4 walls = $(65+15) = 80m^2$

$2(\text{length} + \text{breadth}) * \text{height} = 80$.

Therefore, $2*5x*2x = 80$ or $x^2 = 4$ or $x = 2$

Therefore, height of the room = 4m

11. Ans: b

$\frac{1}{2}(12+8)d = 840$ or $d = 84m$

12. Ans: c

Area = $\sqrt{3}/4 * (3\sqrt{3})^2 = 27\sqrt{3}/4$.

Therefore height = $27\sqrt{3}/4 * 2/3\sqrt{3} = 9/2 = 4.5cm$

13. Ans: b

Let the other sides be x and y . Then,

$x^2 + y^2 = 13^2 = 169$. Also, $\frac{1}{2}xy = 30 \Rightarrow xy = 60$.

Therefore, $(x+y) = \sqrt{(x^2+y^2)+2xy} = \sqrt{169+120} = \sqrt{289} = 17$.

$(x-y) = \sqrt{(x^2+y^2)-2xy} = \sqrt{169-120} = \sqrt{49} = 7$.

Solving $x+y=17$, $x-y=7$, we get $x=12$ and $y=5$.

Therefore, shorter side = 5cm

14. Ans: c

$2\pi r - r = 37$ or $(2\pi - 1)r = 37$.

Or $(2 * 22/7 - 1)r = 37$ or $37r/7 = 37$ or $r = 7$.

Therefore, Area = $\pi r^2 = (22/7 * 7 * 7) = 154 cm^2$

15. Ans: c

$$2\pi(R-r)=60 \Rightarrow 2 \cdot \frac{22}{7} \cdot (R-r)=60.$$

$$\text{Therefore, } (R-r) = \left(\frac{66 \cdot 7}{44}\right) = 10.5\text{m}$$

16. Ans: c

$$\text{Distance moved by toothed wheel in 15 revolutions} = (15 \cdot 2 \cdot \frac{22}{7} \cdot 25)$$

$$\text{Distance moved by smaller wheel in 1 revolution} = (2 \cdot \frac{22}{7} \cdot 15)$$

$$\text{Therefore, required number of revolutions} = \left(\frac{15 \cdot 44 \cdot 7 \cdot 25 \cdot 7}{44 \cdot 15}\right) = 25$$

17. Ans: b

$$\text{Length of wire} = \text{circumference of circle of radius } 42\text{cm} = (2 \cdot \frac{22}{7} \cdot 42) = 264\text{cm}.$$

$$\text{Therefore, perimeter of rectangle} = 264 \text{ cm}.$$

$$\text{Let, length} = 6x \text{ cm \& breadth} = 5x \text{ cm}.$$

$$\text{Therefore, } 2(6x+5x) = 264 \text{ or } x=12.$$

$$\text{Therefore, smaller side} = 60 \text{ cm}$$

18. Ans: d

$$\text{Required area} = (63 \cdot 63 - 4 \cdot \frac{1}{4} \cdot \frac{22}{7} \cdot 63 \cdot \frac{63}{2}) = 850.5\text{m}^2$$

19. Ans: a

$$\text{Volume of theater} = \text{Volume of lower portion} + \text{volume of upper portion}.$$

$$\text{Volume of lower portion (Rectangular prism)} = lbh = 8 \cdot 4 \cdot 25 = 800\text{cm}^3$$

$$\text{Volume of upper portion (triangular prism)} = \left(\frac{1}{2} bh\right)h = \frac{1}{2} \cdot 8 \cdot 3 \cdot 25 = 300 \text{ cm}^3$$

$$\text{Therefore, Total volume} = \text{Volume of lower portion} + \text{volume of upper portion}$$

$$= 800 + 300 = 1100\text{cm}^3$$

20. Ans: a

$$d=12, r=6;$$

$$\text{Volume of the largest sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \cdot \pi \cdot 6 \cdot 6 \cdot 6 = 288\pi\text{cm}^3$$